Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** Mathematical Analysis **12 (2005)** 719-730

ASYMPTOTIC BEHAVIOR OF THE SOLUTION FOR A DIFFUSIVE SYSTEM COUPLED WITH LOCALIZED SOURCES

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Abstract. This paper investigates the positive solution of a diffusive system with localized sources: $u_t - \Delta u = v^p(x_0, t)$, $v_t - \Delta v = u^q(x_0, t)$ in a bounded domain, $p, q \geq 1, pq > 1$. Under appropriate hypotheses, it is proved that the unique classical solution either exists globally or blows up in finite time. The authors also obtain its blow-up set and asymptotic behavior.

Keywords. diffusive system, localized source, blow-up, global existence, blow-up set, asymptotic behavior of the solution.

AMS (MOS) subject classification: 35K57, 35K60.

1 Introduction and main results

In this paper, we investigate the positive solution of a diffusive system coupled with localized sources:

$$\begin{aligned} u_t - \Delta u &= v^p(x_0, t), & v_t - \Delta v &= u^q(x_0, t), & x &\in \Omega, t > 0, \\ u(x,t) &= 0, & v(x,t) &= 0, & x &\in \partial\Omega, t > 0, \\ u(x,0) &= u_0(x), & v(x,0) &= v_0(x), & x &\in \Omega, \end{aligned} \tag{1.1}$$

where $p, q \geq 1, pq > 1$, $\Omega \subset R^N$ is a bounded domain with boundary $\partial\Omega \in C^{2+\gamma}$, $\gamma \in (0,1)$, $x_0 \in \Omega$ is a fixed point.

The equations in (1.1) describe some physical phenomena in which the nonlinear reaction in a dynamical system takes place only at a single site, see [1, 7].

For the scalar more general equation

$$u_t - \Delta u = f(u(x_0, t)), \quad x \in \Omega, \, t > 0, \tag{1.2}$$

it has been studied by many authors, see [2, 3, 4, 8, 10]. Cannon and Yin [3] proved that (1.2) admits a unique classical solution. Chadam, Peire and Yin [2] showed that the solution u blows up in finite time if $u_0(x)$ is sufficiently large in a neighborhood of x_0 and $f(s)$ satisfies: $f(s) \geq 0$, $f'(s) \geq$

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