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RECOVERING MEMORY KERNELS IN RETARDED FUNCTIONAL DIFFERENTIAL EQUATIONS *

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Abstract. This paper is devoted to recovering a scalar memory kernel in an abstract retarded functional differential equation of parabolic type. For such a problem existence, uniqueness and regularity results are proved. Applications to partial differential parabolic equations with delay are given.
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1 Introduction

Let $A : D(A) \subset E \to E$ be the infinitesimal generator of an analytic semigroup in a Banach space E. The aim of this paper consists of recovering the unknown pair (u, a), $u : [0, T] \to E$ and $a : [-r, 0] \to \mathbf{R}$, in the following abstract delay functional differential problem:

$$u'(t) = Au(t) + Au(t-r) + \int_{-r}^{0} a(s)Au(t+s)\,ds + f(t), \quad t \in (0, T), \tag{1.1}$$

$$u(s) = \varphi(s), \ s \in [-r, 0], \tag{1.2}$$

where $T \geq r$ and $f : (0, T) \to E$ and $\varphi : [-r, 0] \to D(A)$, are given.

Since problem (1.1), (1.2) is *underdetermined*, indeed it allows to determine u for a known a, to recover the pair (u, a) we need to prescribe an additional information. A possible choice is the following

$$\Phi[u(t)] = g(t), \qquad t \in [0, r], \tag{1.3}$$

$g : [0, r] \to \mathbf{R}$ and Φ being, respectively, a given function and a linear continuous functional defined on the whole of E.

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