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EXISTENCE RESULT FOR SOME FUNCTIONAL DIFFERENTIAL EQUATIONS WITH INFINITE DELAY

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Abstract: This work is devoted to the existence of bounded solutions for some functional differential equation with infinite delay. The existence of bounded solution is used to prove the existence of periodic solution.

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1 Introduction

The purpose of this work is to study the existence of bounded and periodic solutions of the following functional differential equation

$$\begin{cases} \dfrac{d}{dt}x(t) = F(t, x(t), x_t), \ \text{ for } t \geq 0 \\ x_0 = \varphi \in C = BC\left((-\infty, 0], \mathbb{R}^n\right), \end{cases} \tag{1}$$

where $BC\left((-\infty, 0], \mathbb{R}^n\right)$ is the space of bounded continuous functions from $(-\infty, 0]$ into $\mathbb{R}^n$ provided with the uniform norm topology, it's norm is denoted by $\|.\|$, F is a function from $\mathbb{R}^+ \times \mathbb{R}^n \times C$ into $\mathbb{R}^n$ satisfying some assumptions (see below), and for every $t \geq 0$, the function $x_t \in C$ is defined by

$$x_t(\theta) = x(t + \theta), \ \text{ for } \theta \in (-\infty, 0].$$

The theory of existence of solutions of functional differential equations in infinite delay has been established at first by Hale and Kato [5] and after it has been developed by several authors, for more details we refer to Naito et al [15]. Note that in finite delay case, the phase space is the space of all continuous functions from $[-\tau, 0]$ into E and in that case the theory for such equation is developed by several authors, we refer to Hale et al [4]. In infinite delay case the abstract phase space is not necessarily C, however it's assumed to satisfy some axioms which has been introduced at first Those axioms ensure the existence of solutions for infinite delay differential equations. Note that the space $BC\left((-\infty, 0], \mathbb{R}^n\right)$ doesn't satisfy axiom $(A1)$ see Hale et al [4], p.