

LIMIT CYCLES IN A CLASS OF BIO-CHEMISTRY REACTION SYSTEMS

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Abstract: This paper considers the qualitative analysis of a class of biochemistry reaction system of the form $\dot{x} = a - bx - xy^n$, $\dot{y} = bx - y + xy^n$. Some sufficient conditions for the boundedness of solutions and the existence of limit cycle are obtained.

Keywords: Biochemistry reaction systems; Boundedness of solutions; Limit cycle; Existence.

AMS (MOS) subject classification: 34C05

1 Introduction

A classical problem in the qualitative theory of real planar differential equations is the determination of limit cycles. Limit cycles of planar vector fields were defined by Poincare in the 1880s [1]. At the end of the 1920s van der Pol [2], Lienard [3] and Andronov [4] proved that a closed trajectory of a self-sustained oscillation occurring in a vacuum tube circuit was exactly the same limit cycle as considered by Poincare. After that, the existence, non-existence, uniqueness and other properties of the limit cycles have been studied extensively by mathematicians and physicists, and more recently also by chemists, biologists, economists, etc. See [5-20] and references therein. This paper studies the limit cycles of a class of biochemistry reaction systems.

The earliest reported periodic chemical reaction occurring in homogeneous solution was the decomposition of hydrogen peroxide catalyzed by HIO₃-I₂. The earliest mathematical model to describe "continuous oscillation" was Lotka. Chemical oscillation did not receive much attention since it was regarded as a rare case at that time. Since 1960s, "continuous oscillation" has been found in biochemistry reaction and has become an important part in experimental researches. Meanwhile, in order to explain the functions of ecology and chemical reactors and illustrate the periodic oscillation phenomena in different fields of biology, the mathematical model method has