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TRAVELING WAVE BEHAVIOR FOR A NONLINEAR REACTION-DIFFUSION EQUATION

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Abstract. There is the widespread existence of wave phenomena in physics, chemistry and biology. In the present paper, we study a nonlinear reaction-diffusion equation, which can be regarded as a generalized Fisher equation. Applying the bifurcation theory of planar systems, bifurcations of bell-profile waves and kink-profile waves for the generalized Fisher equation are illustrated under certain parameter conditions. From there, a bounded traveling wave solution is obtained by means of a series of nonlinear coordinate transformations. At the end of the paper, the asymptotic behaviors of proper solutions for the generalized Fisher equation are established by applying the qualitative theory of differential equations.

Keywords: traveling waves, Fisher equation, bifurcation, proper solution, asymptotic behavior. **AMS (MOS) Subject Classification:** 34C05, 34C20, 35B40.

1 Introduction

It is well recognized that a great number of physical, chemical and biological phenomena can be described by nonlinear differential equations. In particular, two classes of nonlinear differential equations have attracted considerable theoretical attention: (i) those arising from completely integrable, infinite-dimensional Hamiltonian systems [1]; (ii) the so-called reaction-diffusion equations, which combine the effects of a local reaction with those of spatial diffusion [2]. For such nonlinear differential equations, construction of particular traveling wave solutions that have physical, chemical or biological interpretation and meanings is one of basic problems in their own scientific areas. Typical examples are provided by the fact that there are many phenomena in biology where a key element or precursor of a developmental process seems to be the appearance of a traveling wave of chemical concentration (or mechanical deformation). When reaction kinetics and diffusion are coupled, traveling waves of chemical concentration can effect a biochemical

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