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NEW STABILITY CRITERIA OF DELAY PARTIAL DIFFERENCE EQUATIONS

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Abstract. This paper is concerned with the following linear delay partial difference equation

$$u(i,j+1) = a(i,j)u(i+1,j) + b(i,j)u(i,j) + p(i,j)u(i-\sigma, j-\tau), \quad i,j \in N_0$$

where σ and τ are nonnegative integers, $\{a(i,j)\}$, $\{b(i,j)\}$ and $\{p(i,j)\}$ are double real sequences. Sufficient conditions for stability and instability of this equation are derived.
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1 Introduction

Recently, there have been many papers that study the qualitative theory of difference equations [1-8]. In particular, the oscillation and frequent oscillation of difference equations are studied in [4,5] and the stability of difference equations is discussed in [2,6-8].

In this paper, we consider the partial difference equation of the form

$$u(i,j+1) = a(i,j)u(i+1,j) + b(i,j)u(i,j) + p(i,j)u(i-\sigma, j-\tau), \quad (1.1)$$

where σ and τ are nonnegative integers, and $\{a(i,j)\}$, $\{b(i,j)\}$ and $\{p(i,j)\}$ are real sequences defined on $i \geq 0$ and $j \geq 0$.

By a solution of Eq.(1.1) we mean a real double sequence $\{u(i,j)\}$ which is defined for $i \geq -\sigma$ and $j \geq -\tau$, and satisfies (1.1) for $i \geq 0$ and $j \geq 0$.

Let t be an integer, $N_t = \{t, t+1, \cdots\}$ and $\Omega = N_{-\sigma} \times N_{-\tau} \backslash N_0 \times N_1$. It is obvious that for any given sequence $\varphi = \{\varphi(i,j)\}$ defined on Ω, it is easy to construct by induction a double sequence $\{u(i,j)\}$ which equals φ on Ω and satisfies (1.1) on $N_0 \times N_1$. The sequence $\{u(i,j)\}$ is said to be a solution of Eq.(1.1) with the initial condition φ.

Stability of Eq.(1.1) has been investigated in [6-8] by several authors. In fact, Lin and Cheng [6] only considered a special case of (1.1) where $p(i,j) = 0$ for any $i,j = 0,1,2\cdots$. Zhang and Tian [7], Zhang and Deng [8] obtained several stability criteria of Eq.(1.1) when $p(i,j) \neq 0$ for some $i,j \in N_0$. In