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ON THE EXISTENCE OF SOLUTIONS TO BOUNDARY VALUE PROBLEMS ON TIME SCALES

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Abstract. This work formulates existence theorems for solutions to two-point boundary value problems on time scales. The methods used include maximum principles, a priori bounds and topological degree theory.

Keywords. time scale, measure chain, boundary value problem, topological degree, dynamic equation.
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1 Introduction

Motivated by the desire to unify the theory of continuous and discrete calculus, Stefan Hilger [11] introduced the theory of time scales in 1990. Hilger defined the "generalized derivative" $y^{\Delta}(t)$, where the domain of the function is a so-called "time scale" (which is an arbitrary closed subset of $\mathbb{R}$). By choosing the time scale to be, say $\mathbb{R}$, then the generalized derivative is just the usual derivative from calculus, ie $y^{\Delta}(t) = y'(t)$. By choosing the time scale to be, say $\mathbb{Z}$, then the generalized derivative is just the usual forward difference, ie $y^{\Delta}(t) = \Delta y(t)$. There are many more time scales than just these two cases.

This paper considers the existence of solutions to the second-order dynamic equation

$$y^{\Delta\Delta}(t) = f(t, y(\sigma(t))), \quad t \in [a, b], \tag{1}$$

subject to the separated boundary conditions

$$g((y(a), y(\sigma^2(b)); (y^{\Delta}(a), y^{\Delta}(\sigma(b)))) = (0, 0), \tag{2}$$

where $f : [a, b] \times \mathbb{R} \to \mathbb{R}$ and t is from a so-called "time scale". Together, equations (1) and (2) are known as a boundary value problem (BVP) on time scales. Once again, by choosing the time scale to be, say $\mathbb{R}$, then (1) will be a second-order, ordinary differential equation. By choosing the time scale to be, say $\mathbb{Z}$, then (1) will be a second-order difference equation. There are many more BVPs on time scales than just these two cases.