Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **12 (2005)** 579-593

LIMITING BEHAVIORS OF DELAY DISCRETE-TIME NEURAL NETWORKS

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Abstract. We consider a nonlinear discrete-time system that describes the computational performance of a discrete-time network of two neurons with delayed feedback given by the McCulloch-Pitts nonlinearity, where $\beta \in (0,1)$, K is a positive integer, f is a signal transmission function with a threshold $\sigma \in R$ and a synaptic weight $\rho > 0$. We concentrate on the cases where $\varphi - \sigma$ and $\psi - \sigma$ have sign changes. We investigate asymptotic behaviors of solutions of the system for the cases where $\sigma \leq -\frac{\rho}{1-\beta}$.
Keywords. Asymptotic behavior, Discrete analog, Sign change, McCulloch-Pitts nonlinearity, Neural network.
AMS (MOS) subject classification: 39A10, 39A12.

1 Introduction

Let Z denote the set of all integers. For any $a, b \in Z$ with $a \leq b$, define $N(a) = \{a, a+1, \cdots\}$, and $N(a,b) = \{a, a+1, \cdots, b\}$. Also, let $N = N(0)$. In this paper, we consider the following nonlinear discrete-time system

$$\begin{cases} x(n) = \beta x(n-1) + f(y(n-K)), \\ y(n) = \beta y(n-1) + f(x(n-K)), \end{cases} \quad n \in N \tag{1.1}$$

where $\beta \in (0,1), K \in N$ and $f : R \to R$ is given by

$$f(x) = \begin{cases} -\rho & \text{if } x > \sigma, \\ \rho & \text{if } x \leq \sigma, \end{cases} \tag{1.2}$$

for two parameters $\rho > 0$ and $\sigma \in R$.

System (1.1) can be regarded as the discrete analog of the following artificial neural network of two neurons with delayed feedback(see[3])

$$\begin{cases} \dot{x} = -\mu x(t) + g(y(t-\tau)), \\ \dot{y} = -\mu y(t) + g(x(t-\tau)), \end{cases} \tag{1.3}$$

where $\dot{x}$ and $\dot{y}$ are replaced by the backward difference $x(n) - x(n-1)$ and $y(n) - y(n-1)$ respectively. System (1.3) has found interesting application in image processing of moving objects [3,5],and has been recently investigated [4]. In the discrete version (1.1), β is the internal decay rate, f is