Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **12 (2005)** 685-705

EXACTLY TWO POSITIVE SOLUTIONS OF NONHOMOGENEOUS SEMILINEAR ELLIPTIC EQUATIONS IN UNBOUNDED CYLINDER DOMAINS

Tsing-San Hsu

Center of General Education
Chang Gung University, Kwei-San, Tao-Yuan 333, Taiwan

Abstract. In this paper, we consider the nonhomogeneous semilinear elliptic equation

$$-\Delta u + u = \lambda K(x)u^p + h(x) \text{ in } \Omega, u > 0 \text{ in } \Omega \text{ }, u \in H_0^1(\Omega), \qquad (*)_\lambda$$

where $\lambda \geq 0$, $N \geq 3$, $1 < p < N+2/N-2$, and Ω is an unbounded cylinder domain. Under some suitable conditions on K and h, we show that there exists a positive constant λ^* such that $(*)_\lambda$ has exactly two solutions if $\lambda \in (0, \lambda^*)$ and no solution if $\lambda > \lambda^*$. Furthermore, $(*)_\lambda$ has at least one solution for $\lambda = \lambda^*$ provided that $h(x) \in L^{\frac{2N}{N+2}}(\Omega) \cap L^\infty(\Omega)$.

Keywords. nonhomogeneous, elliptic equation, unbounded cylinder, minimal solutions.
AMS (MOS) subject classification: 35J20, 35J25, 35J60.

1 Introduction

In this paper, we consider the semilinear elliptic equation

$$\begin{cases} -\Delta u + u = \lambda K(x)u^p + h(x) \text{ in } \Omega, \\ u > 0 \text{ in } \Omega \text{ }, u \in H_0^1(\Omega), \end{cases} \qquad (1.1)_\lambda$$

where $\lambda \geq 0$, $N = m+n \geq 3$, $m \geq 2$, $n \geq 1$, $1 < p < N+2/N-2$, $0 \in \omega \subseteq \mathbb{R}^m$ a bounded $C^{1,1}$ domain, $\Omega = \omega \times \mathbb{R}^n$, $h(x) \in H^{-1}(\Omega)$, $0 \not\equiv h(x) \geq 0$ in Ω, $K(x)$ is a positive, bounded and continuous function on $\overline{\Omega}$. Moreover, $K(x)$ satisfies assumption (H) below.

(H) $K(x) \geq K_\infty > 0$ in $\overline{\Omega}$, and

$$\lim_{|z|\to\infty} K(x) = K_\infty \text{ uniformly for } y \in \overline{\omega}.$$

If Ω is bounded ($n = 0$ in our case), the equation $(1.1)_\lambda$ has been studied by many authors : see for instance Bahri-Lions [3] and the references therein. We only consider that Ω is unbounded ($n \geq 1$ in our case). If $\Omega = \mathbb{R}^N$ ($m = 0$ in our case), Zhu [17], Zhu-Zhou [19] and Cao-Zhou [7], established the existence of multiple positive solutions of equations with structure unlike that here.