*Dynamics of Continuous, Discrete and Impulsive Systems*
*Series* **A:** *Mathematical Analysis* **12 (2005)** 563-578


# EXISTENCE OF SOLUTIONS OF DIFFUSIVE LOGISTIC EQUATIONS WITH IMPULSES AND TIME DELAY AND STABILITY OF THE STEADY-STATES

Jalina Widjaja and Murk J. Bottema

School of Informatics and Engineering
Flinders University, Adelaide, South Australia SA 5001

**Abstract.** The existence of solution of diffusive logistic equation with impulses and time delay is proved using upper and lower solutions. The timing of impulses in this system are fixed. Conditions are found under which the zero function and the steady-state solution to the problem without impulses are attractors.
**Keywords.** Delay, impulse, logistic, diffusion, upper and lower solutions.
**AMS (MOS) subject classification:** 35B35,35R12,35R10,92D25.

## 1 Introduction

Many processes, both natural and man-made, in biology, medicine, chemistry, physics, engineering, economics, and so forth, involve time delay. Some of these processes are also characterized by the fact that the system parameters are subject to short-term perturbation in time. An adequate apparatus for mathematical simulation of such processes and phenomena is impulsive differential equations with time delays.

The logistics equation is a simple nonlinear equation that is famous for its role in population studies but appears in all the applications mentioned above. Many researchers have investigated the logistic equation with one or two of these phenomena: impulse, delay, diffusion. The local and global existence of solution of logistic equation with impulses are shown in [13]. Under certain conditions, the uniqueness of the solution is proved. By linear approximation, it is shown that the trivial solution is stable. The existence, uniqueness, and stability of solution of this problem are also discussed in [14], from a distributional approach.

Du and Ma [6] presented the existence and uniqueness of the positive solution of logistic equation with diffusion. They also showed that the positive steady-state was stable.

For the logistic equation with delay and impulses, Yu and Zhang in [23] used the results in [22] to find sufficient conditions for the stability of the zero solution. The stability (including uniform stability and uniform asymptotic stability) of the zero solution has been investigated by Bainov et al in [2]