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Towards a Geometric Theory of Hybrid Systems

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Abstract. We propose a framework for a geometric theory of hybrid systems. Given a deterministic, non-blocking hybrid system, we introduce the notion of its hybrifold with the associated hybrid flow on it. This enables us to study hybrid systems from a global geometric perspective as (generally non-smooth) dynamical systems. This point of view is adopted in studying the Zeno phenomenon. We show that it is due to nonsmoothness of the hybrid flow. We introduce the notion of topological equivalence of hybrid systems and locally classify isolated Zeno states in dimension two.
Keywords. Hybrid system; dynamical system; hybrifold; Zeno; topological equivalence.
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1 Introduction

Any system which changes in time and involves interactions between continuous processes and discrete automata can be viewed as a hybrid system. The name comes from the hybrid nature of its evolution: intervals of continuous-time change are interleaved with instantaneous, discrete jumps. Hybrid systems are a topic of great interest, primarily in the control and computer science communities. Emerging applications in areas such as air traffic management [37], automotive control [5, 1], real-time software verification [2], transportation systems [39, 25], manufacturing [32], mobile robotics [6], and process industry [13], motivate work on hybrid systems modeling [11, 28, 4, 38, 10, 20], analysis [3, 40, 9, 22, 17, 23], and controller design [29, 8, 10, 27, 7]. Although extensive research efforts have been made in the area of hybrid systems, the understanding of the fundamental dynamical properties of systems with interacting continuous-time and discrete-event dynamics [11, 20, 16, 21, 42] is not satisfactory.