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ARBITRAGE OPPORTUNITIES ON DERIVATIVES: A LINEAR PROGRAMMING APPROACH

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Abstract. I propose a simple tool to determine the existence of arbitrage opportunities on derivatives with the same underlying asset and maturity, when only minimal assumptions on the stochastic behavior of the underlying are made. The case of transaction costs is also considered. The use of linear programming theory makes the approach very easily implementable.

Necessary and sufficient conditions for no arbitrage, as well as an application on options written on the S&P500 index are given.

Keywords. Linear programming, option pricing, arbitrage, financial derivatives

AMS (MOS) subject classification: 90C90

1 Introduction

The purpose of this paper is to determine necessary and sufficient conditions for detecting static arbitrage opportunities on a set of derivative securities written on the same underlying S and with the same maturity T. The key question posed here is: "Given some prices of derivatives, is there a simple way to check for arbitrage opportunities?". Indeed, the theoretical answer to such question is well known: "There are no arbitrages if and only is there exists an equivalent martingale measure!". However, this may be not so useful in practice, as equivalent martingale measures may be not readily available. Moreover, such an answer relies on various assumptions on the probabilistic behavior of the underlying asset. My goal is to develop a simple tool, suited for practical applications, making as few assumptions as possible.

For a formal definition of arbitrage one can look at standard references like Bjork [2], Duffie [5] and many others. Usually, arbitrages are classified in two different types: A and B. An arbitrages of *type A* is a financial strategy where one enters for free, no liabilities will ever be claimed, and there is a positive probability of some positive income in the future (a free lottery ticket is an example of this). In an arbitrage of *type B* there is a positive cash-flow at time 0 and no liabilities in the future. Arbitrages may also be classified as *dynamic* and *static*. A dynamic arbitrage is a strategy that re-balances the portfolio following the evolution of market prices, while in a static arbitrage, the portfolio remains unchanged.

I want to state arbitrage conditions that are independent (as far as possible) from a particular pricing model. For this reason, I will not take into