

## PORTFOLIO REBALANCING WITH TRANSACTION COSTS AND A MINIMAL PURCHASE UNIT

Y. Fang<sup>1</sup>, K.K. Lai<sup>2</sup> and S.Y. Wang<sup>1</sup>

<sup>1</sup>Institute of Systems Science, Academy of Mathematics and Systems Sciences  
Chinese Academy of Sciences, Beijing 100080, China

<sup>2</sup>Department of Management Sciences  
City University of Hong Kong, Kowloon, Hong Kong

**Abstract.** A mean semi-absolute deviation model is proposed for portfolio rebalancing with transaction costs and taxes. Considering the existence of a minimal purchase unit of securities, a mixed integer linear programming model is proposed. Due to the high computational complexity of the model, a heuristic algorithm is proposed. An example is given to illustrate that the model and the heuristic algorithm can be used efficiently to solve portfolio rebalancing problem by using real data from the Shanghai Stock Exchange.

**Keywords.** Portfolio rebalancing, transaction costs, minimal purchase unit, semi-absolute deviation risk function, mixed-integer linear programming problem.

**AMS (MOS) subject classification:** 91B28, 90B50, 90C11, 90C90

## 1 Introduction

Originally proposed by Markowitz in [11], the mean variance methodology for portfolio selection has been central to research activities in this area and has served as a basis for the development of modern financial theory over the past five decades. However, Markowitz's standard model, contrary to its theoretical reputation, is not used extensively to construct large scale portfolios. One of the most important reasons is the computational difficulty associated with solving a large-scale quadratic programming problem with a dense covariance matrix. Konno and Yamazaki [8] used a piece-wise linear risk function, *i.e.*, absolute deviation risk function, to replace the risk function in Markowitz's model and formulated a mean absolute deviation portfolio selection model. The model can realize the intention of Markowitz's model by solving a linear program instead of any quadratic programming problem. It turns out that the mean absolute deviation model maintains the nice properties of Markowitz's model and removes most of the principal difficulties in solving Markowitz's model. Simaan [17] provided a thorough comparison of the mean variance model and the mean absolute deviation model. Furthermore, Speranza [18] and Mansini and Speranza [10] used the semi-absolute deviation to measure the risk and formulated a mean semi-absolute deviation portfolio selection model.