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Portfolio Optimization under Long-Short Constraints

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Abstract. This paper is concerned with a special and yet important class of practical portfolio optimization problems. A fund manager is supposed to use up a fixed amount of fund supplied by the client at the time of portfolio construction. When he purchases some assets, the cash outflow can be represented as a linear function of the amount to be purchased. When he sells the asset short, the net cashflow is more complicated. The cash obtained by the short sale will be temporarily held at the third party who lends assets until this short sale is cleared. Also the fund manager has to pay certain amount of deposit and commission fee to the third party. Therefore, the net cash outflow becomes a nonconvex function of the amount of the investment into each asset.

We will look into the special structure of this long-short portfolio optimization problem and propose a branch and bound algorithm. It will be demonstrated that this algorithm can solve virtually all test problems in a very efficient manner.

Keywords. Portfolio optimization, short sale, nonconvex constraint, global optimization problem, branch and bound method.

1 Introduction

Let us consider a fund manager who invests the client's money of amount M into a number of assets using mean-variance model [8]. He will either

(i) minimize the variance $V[R(\boldsymbol{x})]$ of the return $R(\boldsymbol{x})$ of the portfolio $\boldsymbol{x}$ over the investable set $\boldsymbol{X} \subset R^n$ subject to the constraint that the expected rate of return $E[R(\boldsymbol{x})]$ is equal to a given constant ρ specified by the client,

or

(ii) maximize the risk adjusted return $E[R(\boldsymbol{x})] - \lambda V[R(\boldsymbol{x})]$ over the investable set $\boldsymbol{X}$, where $\lambda \geq 0$ is a constant representing the degree of risk averseness of the client.

It is well known that these two models are equivalent in the sense that they generate the same efficient frontier as we vary ρ in (i) and λ in (ii). It is usually assumed here that no assets are allowed to sell short, *i.e.*,$\boldsymbol{x} \geq \boldsymbol{0}$. The investable set may then be represented as follows (ignoring the transaction costs):