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FORMATION OF SINGULARITIES IN SOLUTIONS OF A QUASILINEAR STRICTLY HYPERBOLIC SYSTEM

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Abstract. We consider a special type of strictly hyperbolic systems and show that the gradient of the solution develops singularities in finite time.
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1 Introduction.

In a previous work [5], we considered the following one-dimensional quasilinear wave equation

$$w_{tt}(x,t) = \sigma\left(\frac{w_t(x,t)}{w_x(x,t)}\right) w_{xx}(x,t) \tag{1}$$

and showed that, for well chosen initial data, the classical solutions develop singularities in finite time. In the present work we prove a similar result for a strictly hyperbolic system, which can be regarded as a relative generalization of (1.1). More precisely we study the system

$$\begin{cases} u_t(x,t) = a\left(\dfrac{u(x,t)}{v(x,t)}\right) v_x(x,t) \\ v_t(x,t) = b\left(\dfrac{u(x,t)}{v(x,t)}\right) u_x(x,t) \end{cases} \tag{2}$$

where a subscript denotes a partial derivative with respect to the relevant variable; $x \in I = (0,1)$, and $t > 0$.

It is well known that, generally, classical solutions for hyperbolic systems break down in finite time even for smooth and small initial data. For instance Lax [7] and MacCamy and Mizel [12] studied the system for a depending on v only and $b \equiv 1$. They showed that classical solutions blow up in finite time even if the initial data are smooth and small. In his work Lax required that $a^{'} > 0$; whereas MacCamy and Mizel allowed $a^{'}$ to change sign. Note that, in this particular case, the system is reduced to the well known nonlinear wave equation. For systems with dissipation the situation is different. If the