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NON-ORDERED LOWER AND UPPER FUNCTIONS IN SECOND ORDER IMPULSIVE PERIODIC PROBLEMS

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Abstract. In this paper, using the lower/upper functions argument, we establish new existence results for the nonlinear impulsive periodic boundary value problem

$$u'' = f(t, u, u'), \tag{1.1}$$

$$u(t_i+) = \mathcal{J}_i(u(t_i)), \quad u'(t_i+) = \mathcal{M}_i(u'(t_i)), \quad i = 1, 2, \ldots, m, \tag{1.2}$$

$$u(0) = u(T), \quad u'(0) = u'(T), \tag{1.3}$$

where $f \in \mathrm{Car}([0,T] \times \mathbb{R}^2)$ and $\mathcal{J}_i$, $\mathcal{M}_i \in \mathbb{C}(\mathbb{R})$. The main goal of the paper is to obtain the results in the case that the lower/upper functions σ_1/σ_2 associated with the problem are not well-ordered, i.e. $\sigma_1 \not\leq \sigma_2$ on $[0,T]$.

Keywords. Second order nonlinear ordinary differential equation with impulses, periodic solutions, lower and upper functions, Leray-Schauder topological degree, a priori estimates.
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1 Introduction

In this paper we provide new conditions for f, $\mathcal{J}_i$, $\mathcal{M}_i$, $i = 1, 2, \ldots, m$, which guarantee the existence of a solution of the nonlinear impulsive periodic boundary value problem (2.1)–(2.3). We have studied this problem in [11] using arguments based on the existence of a well-ordered pair $\sigma_1 \leq \sigma_2$ on $[0,T]$ of lower/upper functions σ_1/σ_2 associated with the problem. Such assumption corresponds to requirements imposed by Hu Shouchuan and Lakshmikantham [6] (see also Bainov and Simeonov [1]), Erbe and Liu Xinzhi [5], Liz and Nieto [7], [8], Dong Yujun [4] and Zhang Zhitao [12] who have investigated the problems of the type (2.1)–(2.3). Note that a similar problem with different impulse conditions was recently treated by Cabada, Nieto, Franco and Trofimchuk [2]. However, their principal assumption was that of the existence of well-ordered pair of lower/upper functions, as well.

Here, we consider problem (2.1)–(2.3) in a more complicated case. Particularly, we assume that there are only lower/upper functions to (2.1)–(2.3)