Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **12 (2005)** 383-396

UPPER AND LOWER SOLUTIONS METHOD FOR IMPULSIVE DIFFERENTIAL INCLUSIONS WITH NONLINEAR BOUNDARY CONDITIONS AND VARIABLE TIMES

M. Benchohra[1], J. R. Graef[2], S. K. Ntouyas[3], and A. Ouahab[1]

[1]Department of Mathematics
University of Sidi Bel Abbes, BP 89 2000 Sidi Bel Abbes, Algeria
[2]Department of Mathematics
University of Tennessee at Chattanooga, Chattanooga, TN 37403-2504 USA
[3]Department of Mathematics
University of Ioannina, 451 10 Ioannina, Greece

Abstract. In this paper, a nonlinear alternative of the Leray Schauder type for multi-valued maps combined with upper and lower solutions are used to investigate the existence of solutions for first order impulsive differential inclusions with nonlinear boundary conditions and variable moments.
Keywords. Boundary value problem, convex multi-valued map, impulsive differential inclusions, nonlinear boundary value problem, fixed point, upper and lower solutions, variable times.
AMS (MOS) subject classification: 34A37, 34A60

1 Introduction

This paper is concerned with the existence of solutions for the first order boundary value problem with impulsive effects

$$y'(t) \in F(t, y(t)), \quad t \in J = [0, T], \quad t \neq \tau_k(y(t)), \quad k = 1, \dots, m, \tag{1}$$

$$y(t_k^+) = I_k(y(t_k^-)), \quad t = \tau_k(y(t)), \quad k = 1, \dots, m, \tag{2}$$

$$L(y(0), y(T)) = 0, \tag{3}$$

where $F : J \times \mathbb{R} \longrightarrow 2^{\mathbb{R}}$ is a compact convex valued multi-valued map and $L : \mathbb{R}^2 \to \mathbb{R}$ is a single-valued map, $\tau_k : \mathbb{R} \to \mathbb{R}$ and $I_k \in C(\mathbb{R}, \mathbb{R})$ $(k = 1, 2, \dots, m)$, are bounded maps, and $y(t^-)$ and $y(t^+)$ represent the left and right hand limits of $y(s)$ at $s = t$, respectively.

The method of upper and lower solutions has been successfully applied to study the existence of solutions for initial and boundary value problems of the first order. This method has been used only in the context of single-valued impulsive differential equations with fixed and/or variable moments. In this regard, we refer the reader to the monographs by Lakshmikantham,