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ON THE BLOW-UP OF SOLUTIONS OF THE PERIODIC CAMASSA-HOLM EQUATION

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Abstract. We present a blow-up result for the periodic Camassa-Holm equation. The obtained result improves recent results.
Keywords. Blow-up of solutions, Camassa-Holm equation, Sobolev inequalities, the time evolution of the extrema of a function, an explosion criterion.
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1 Introduction

In the paper, we discuss the periodic Camassa-Holm equation

$$\begin{cases} u_t - u_{txx} + 3uu_x = 2u_xu_{xx} + uu_{xxx}, & t > 0,\ x \in \mathbb{R}, \\ u(0,x) = u_0(x), & x \in \mathbb{R}, \\ u(t,x+1) = u(t,x), & t \geq 0,\ x \in \mathbb{R}. \end{cases} \tag{1.1}$$

Eq.(1.1) was derived physically by Camassa-Holm [1] as a model for the unidirectional propagation of shallow water waves over a flat bottom, $u(t,x)$ standing for the fluid velocity at time $t \geq 0$ in the spatial x direction (see [1]). The Camassa-Holm equation has a bi-Hamiltonian structure [12] and is completely integrable [9]. The equation is also a re$-$expression of geodesic flow on the diffeomorphism group of the circle [8]. The Cauchy problem of the Camassa-Holm equation has been extensively studied cf. [2, 4, 10, 13, 14, 15] . The equation has global solutions [4, 5] and also solutions which blow-up in finite time [2, 3, 4, 5, 6].

The purpose of the present paper is to study the blow-up phenomenon of Eq.(1.1). Our method relies heavily on some new Sobolev inequalities that are established in Section 2. By applying recent result for the time evolution of the extrema of a function [7] and looking at the time-evolution of the minimum of the slope of a solution, we prove that there exist some solutions of Eq.(1.1) which, while staying bounded, develop singularities in finite time. The obtained result improves recent results in [3, 4, 5].

Our paper is organized as follows. In Section 2, we prove some Sobolev inequalities. In the last section, an explosion criterion for solutions to Eq.(1.1) is given.