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POSITIVE SOLUTIONS FOR A CLASS OF SINGULAR DIFFERENTIAL EQUATIONS ARISING IN DIFFUSION PROCESSES [1]

Alberto Cabada, J. Ángel Cid, and Rodrigo López Pouso

Departamento de Análise Matemática,
Facultade de Matemáticas, Campus Sur,
Universidade de Santiago de Compostela,
15782 Santiago de Compostela, Spain.

E-mail: cabada@usc.es, joseangel.cid@rai.usc.es, rodrigolp@usc.es

Abstract. We derive sufficient conditions for the existence of extremal solutions for a second order singular ordinary differential equation subject to initial data. The type of equations that we study here can be regarded as stationary and one dimensional models for diffusion processes in which the diffusion coefficient is not a constant. We have also tried to relax the regularity assumptions as far as possible, in order to extend the applicability of our result.

Keywords. Diffusion equations, Discontinuous differential equations.
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1 Introduction

A general form of the heat equation is given by

$$\frac{\partial u}{\partial t} - \operatorname{div}(k\nabla u) = f(t, x, u, \nabla u),\ t > 0,\ x \in \Omega \subset \mathbb{R}^n,$$

and it serves as a mathematical model for the distribution of temperatures or for the concentration of a certain substance diffusing in a domain Ω. The value k is the so–called *diffusion coefficient* and it is usually assumed to be a constant. However, strictly speaking from a physical point of view, k may vary along the medium and depend also on the concentration u (see [5]).

Considering that k depends on the concentration the corresponding stationary and one–dimensional model is furnished by ordinary differential equations of the form

$$(k(u)u')'(x) = f(x, u(x), u'(x)).$$

In fact, the nonlinear differential equation

$$(u^{m+1})''(x) + (x-1)u'(x) = 0, \quad x \in (0,1), m > 0, \tag{1}$$

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