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# INTERVAL CRITERIA OF OSCILLATION FOR FORCED SUPERLINEAR DIFFERENCE EQUATIONS[1]

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**Abstract.** Using Riccati transformation techniques, we establish interval criteria of oscillation for forced second-order superlinear difference equations, which are different from the most known ones in the sense that they are based on the information only on a subsequence of $\mathbb{N}$. Our criteria are discrete analogues of the criteria used for differential equations by Kong [4].
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# 1 Introduction

Consider the forced second-order nonlinear difference equation

$$\Delta^2 x_{n-1} + q_n x_n^\gamma = g_n, \tag{1.1}$$

where $\gamma$ is quotient of positive odd integers, $n$ is an integer in the set $\mathbb{N} = \{1, 2 \dots\}$, $\{q_n\}_{n=1}^\infty$ and $\{g_n\}_{n=1}^\infty$ are sequences of real numbers, $\Delta$ denotes the forward difference operator $\Delta x_n = x_{n+1} - x_n$ and $\Delta^2 x_n = \Delta(\Delta x_n)$. In the case $\gamma > 1$, (1.1) is the prototype of a wide class of nonlinear difference equations called Emden-Fowler superlinear difference equations.

In recent years there has been an increasing interest in the asymptotic behavior and oscillatory properties of second-order difference equations, see, e.g., the monographs [1, 2]. Following this trend, we study the oscillations of (1.1). It is interesting to study (1.1) because, it is the discrete version of the second order Emden-Fowler differential equation that has several physical applications (see [10] for details).

We consider only nontrivial solutions of (1.1); i.e., solutions such that for every $i \in \mathbb{N}$, $\sup\{|x_n| : n \geq i\} > 0$. A solution $\{x_n\}$ of (1.1) is said to be oscillatory if for every $n_1 \geq 1$ there exists an $n \geq n_1$ such that $x_n x_{n+1} \leq 0$, otherwise it is non-oscillatory.

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