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# On Positive Fixed Points of Countably Condensing Mappings

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**Abstract.** In this paper, we study the following equation

$$x = Tx$$

in a cone $P$ of a Banach space $X$, where $T : \overline{\Omega} \cap P \to X$ is a countably condensing mapping, $\Omega$ is an open bounded subset of $X$. Existence results are obtained under suitable boundary conditions, and our results generalize the corresponding results in Petryshyn [16].

**Key Words:** Fixed point, countably condensing mapping, countably $k$-set contraction, quasi-normal cone, fixed point index.
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# 1 Introduction

Let $X$ be a real Banach space and $T : D(T) \subseteq X \to X$ be a nonlinear mapping. $T$ is said to be *countably condensing* (respectively, *countably $k$-set contraction*) if

$$\alpha(T(E)) < \alpha(E) \quad \text{(respectively, } \alpha(T(E)) \leq k\alpha(E)),$$

where $k > 0$ is a constant for any countably bounded subset $E$ of $D(T)$ satisfying $\alpha(E) \neq 0$, where $\alpha(\cdot)$ is the Kuratowskii measure of non-compactness. We refer the reader to [1], [4], [5], [10] and [17] for a discussion of countably condensing mappings.

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