Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **12 (2005)** 505-517

POLYNOMIAL NORMAL FORMS FOR 1-RESONANT VECTOR FIELDS WITH MULTIPLE EIGENVALUES

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Abstract. In this paper we study a class of 1-resonant vector fields which may have multiple eigenvalues. We give the simplest normal forms and the index of finite determinacy. For such a vector field having one zero eigenvalue, the result implies that the sub-vector field formed by the hyperbolic variables depends quadratically on the central variable.
Keywords. vector fields, index of finite determinacy, resonance, the simplest normal form, quasi-strongly 1-resonance.
AMS (MOS) subject classification: 34K17, 37G05.

1 Introduction and Main Results

This paper studies the simplest normal forms of a class of finitely determined C^∞ vector fields given by the ordinary differential equation

$$\dot{x} = Ax + f(x), \quad f(0) = 0, \quad x \in \mathbb{R}^n, \tag{1}$$

where f is a smooth vector function without linear terms, and A is an $n \times n$ matrix whose eigenvalues $\lambda := (\lambda_1, \dots, \lambda_n)$ are assumed to be *1-resonant*, namely, the number of generators of the semigroup

$$\{(K, \lambda) = 0, \quad K \in \mathbb{Z}_+^n\} \tag{2}$$

is one. In this paper we shall call such vector fields 1-resonant vector fields (see [1] p55). Examples of 1-resonant vector fields include those cases, say, A has a zero or a pair of purely imaginary eigenvalues while the remaining eigenvalues are generic.

For a given germ of a vector field, to obtain its normal form is a very classical problem. It seeks changes of coordinates which reduce the vector field as much as possible. The distinguished Poincaré-Dulac theorem (see [1]) says that vector field (1) can be formally reduced to its resonant normal form. In other words, the matrix A can be put into the Jordan canonical

[1]Supported by NSFC-10271006