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VARIATIONAL RESULTS ON THE NONLINEAR WAVE EQUATION WITH JUMPING NONLINEARITY

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ABSTRACT: We investigate the multiplicity of the periodic solutions of the nonlinear wave equation with Dirichlet boundary condition, $u_{tt} - u_{xx} + bu^+ - au^- = se_1^+$, where $u^+ = \max\{u, 0\}$, $u^- = \min\{u, 0\}$, $s \neq 0$, $s \in R$ and e_1^+ is the eigenfunction corresponding to the first positive eigenvalue μ_1^+. By critical point theory we reveal that under some conditions the jumping problem has at least $2k+1$ solutions for $s > 0$ and also $2k+1$ solutions for $s < 0$, respectively.
Keywords. Nonlinear wave equation, multiplicity of solutions, Dirichlet boundary condition, linking theorem, eigenvalue
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1 INTRODUCTION

In this paper we investigate the multiplicity of the periodic solutions of the nonlinear wave equation with Dirichlet boundary condition

$$u_{tt} - u_{xx} + bu^+ - au^- = se_1^+ \qquad \text{in } (-\frac{\pi}{2}, \frac{\pi}{2}) \times R, \tag{1.1}$$

$$u(\pm\frac{\pi}{2}, t) = 0,$$

$$u(x,t) = u(-x,t) = u(x,-t) = u(-x, t+\pi),$$

where $u^+ = \max\{u, 0\}$, $u^- = -\min\{u, 0\}$, $s \neq 0$, $s \in R$ and e_1^+ is the eigenfunction corresponding to the positive eigenvalue $\mu_1^+ = 1$ of the eigenvalue problem $u_{tt} - u_{xx} = \mu u$ with Dirichlet boundary condition. Here the terms bu^+ and au^- represent upward and downward restoring forces respectively, due to the vibration of a string with a nonuniform density. In this paper we look for π- periodic solutions of (1.1). Problem (1.1) has the solution $u = \frac{s\phi_{00}}{1+b}$ if $s > 0$, and $u = \frac{s\phi_{00}}{1+a}$ if $s < 0$. Choi and Jung proved in [4] that if $-5 < a < -1, 3 < b < 7$ and $s > 0$, then (1.1) has at least four solutions. In this paper we improved the results of [3] and [4] when the jumping nonlinearity conditions are $-\mu_i^- < a < -\mu_{i+1}^-, \ldots, -\mu_{i+k}^- < b < -\mu_{i+k+1}^-$ and