Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **12 (2005)** 469-477

MAXIMUM PRINCIPLE FOR INTEGRAL DYNAMIC MODELS WITH ENDOGENOUS DELAY

N. Hritonenko [1] and Yu. Yatsenko[2]

[1]Department of Mathematics
Prairie View A&M University,
Prairie View, TX 77446
[2]College of Business and Economics
Houston Baptist University, Houston, TX 77074

Abstract. We investigate optimization problems for non-linear integral equations with unknown functions in the limits of integration. Such problems describe the optimal duration of delay in age-dependent dynamic processes of engineering, economics, and ecology. Necessary conditions for an extremum are examined and a maximum principle is proven.
Keywords. Maximum principle, Volterra integral equations, endogenous delay, optimality conditions, vintage capital models.
AMS (MOS) subject classification: 45D05, 49K22, 93C95.

1 Introduction

Nonlinear Volterra integral equations (VIEs) efficiently describe dynamic processes with delay in engineering, economics, and ecology [2, 5]. The rational renovation of equipment under technological progress leads to optimization problems for Volterra-type integral *models with endogenous delay* [8-10, 19, 20]. Similar problems also arise in rational harvesting of age-structured biological populations and some other applications [7, 9]. Investigation techniques for such problems are provided by the general optimization theory [11, 13-17]. Optimal control of dynamic systems governed by standard VIEs is investigated in [1, 3-5, 12, 16, 18] where various variation techniques are offered for such systems. A new feature of the models with endogenous delay is the presence of unknown functions in the integration limits. Optimization problems for these models are investigated in [7, 10, 19, 20] and others. In this paper, a maximum principle is proven for the integral dynamic models with endogenous delay.

2 Formulation of Optimization Problem

Let us consider the following optimization problem (OP) for Volterra integral equations with *unknowns* in the limits of integration [9, 20]: find