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OSCILLATION AND GLOBAL ATTRACTIVITY OF A PERIODIC SURVIVAL RED BLOOD CELLS MODEL

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Abstract. In this paper, we shall consider the nonlinear delay differential equation

$$N'(t) = -\delta(t)N(t) + P(t)e^{-aN(t-m\omega)}, \tag{$*$}$$

where m is a positive integer, $\delta(t)$ and $P(t)$ are positive periodic functions of period ω. In the nondelay case we shall show that $(*)$ has a unique positive periodic solution $\overline{N}(t)$, and prove that $\overline{N}(t)$ is a global attractor of all other positive solutions. In the delay case we shall present sufficient conditions for the oscillation of all positive solutions of $(*)$ about $\overline{N}(t)$, and establish sufficient conditions for the global attractivity of $\overline{N}(t)$.

Keywords and Phrases: Oscillation, global attractivity, nonlinear delay differential equation, blood cells model.

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1 Introduction

The nonlinear delay differential equation

$$N'(t) = -\delta N(t) + Pe^{-aN(t-\tau)}, \tag{1.1}$$

where

$$\delta,\ P,\ a,\ \tau \in (0,\infty) \tag{1.2}$$

has been proposed by Wazewska–Czyzewska and Lasota [27] to describe the survival of red blood cells in an animal. Here $N(t)$ denotes the number of red blood cells at time t, δ is the probability of death of a red blood cell, P and a are positive constants related to the production of red blood cells per unite of time and τ is the time needed to produce a red blood cell. We also note that several other similar interesting models occurring in population dynamics have been studied in [4,10,13,15,21,22].

Together with (1.1) we shall consider the initial condition

$$N(t) = \phi(t), \quad -\tau \le t \le 0, \quad \phi(0) > 0, \quad \phi \in C([-\tau, 0], R^+). \tag{1.3}$$