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IDENTIFICATION PROBLEMS FOR SINGULAR INTEGRO-DIFFERENTIAL EQUATIONS OF PARABOLIC TYPE I *

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Abstract. We recover unknown kernels, depending on time only, in linear singular first-order integro-differential Cauchy problems in Banach spaces. Singular means here that the integro-differential equation is *not* in normal form neither can it be reduced to such a form. For this class of problems we prove local and global in time existence and uniqueness theorems strictly related to the regularity results proved in [4] for the direct problem. Moreover, we give several applications to explicit singular partial integro-differential equations of parabolic type.
Keywords. Identifying unknown kernels. Abstract linear singular first-order integro-differential equations. Existence and uniqueness results. Linear singular partial integro-differential equations of parabolic type.
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1 Introduction

In this paper we will be concerned with the problem of recovering the kernel k in the following integro-differential Cauchy problem related to the complex Banach space X, with norm $\|\cdot\|$:

$$MD_t u(t) + Lu(t) = \int_0^t k(t-s)L_1 u(s)\,\mathrm{d}s + f(t), \qquad 0 \le t \le \tau, \quad (1.1)$$

$$u(0) = u_0. \quad (1.2)$$

We assume that L, L_1, M are *closed* linear operators from X into itself, with M being *not* necessarily *invertible*, whose domains are related by the relationship $\mathcal{D}(L) \subseteq \mathcal{D}(L_1) \cap \mathcal{D}(M)$. Moreover, we assume that L *admits a continuous inverse operator.* Hence $T = ML^{-1} \in \mathcal{L}(X)$, the space of all bounded linear operators from X into itself, endowed with the uniform norm.

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