Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **12 (2005)** 203-214

MONOTONE APPROXIMATION FOR A HIERARCHICAL AGE-STRUCTURED POPULATION MODEL

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Abstract. We study a nonlinear hierarchical age-structured population model with time dependent individual vital rates. We establish a comparison principle and construct monotone sequences to show the existence and uniqueness of the solution to the model. We also provide conditions on the model parameters which result in extinction or persistence of the population.

Keywords. Hierarchical age-structured population model, monotone approximation, existence-uniqueness, extinction, persistence.

AMS (MOS) subject classification: 35A07, 35B40, 35F30, 35L60, 92D25

1 Introduction

In this paper, we study the following hierarchical age-structured population model:

$$\begin{aligned} &u_t + u_a = -m(a,t,Q(u)(a,t))u, && 0 < a < \infty, \quad t > 0, \\ &u(0,t) = \int_0^\infty \beta(a,t,Q(u)(a,t))u(a,t)da, && t > 0, \\ &u(a,0) = u_0(a), && 0 \le a < \infty. \end{aligned} \tag{1.1}$$

Here $u(a,t)$ is the density distribution of a population of age a at time t, and $Q(u)(a,t)$ is a function of the density u, the environment, given by

$$Q(u)(a,t) = \alpha \int_0^a u(\sigma,t)d\sigma + \int_a^\infty u(\sigma,t)d\sigma, \qquad 0 \le \alpha < 1. \tag{1.2}$$

The functions $m(a,t,Q(u)(a,t))$ and $\beta(a,t,Q(u)(a,t))$ represent, respectively, the mortality and fertility rates of an individual of age a at time t which also depend on the environment.

Age-structured population models have been extensively studied over the past three decades (e.g., [3, 5, 6, 7, 10, 13]), and most investigations have