

GENERALIZED QUASILINEARIZATION METHOD FOR A FIRST ORDER DIFFERENTIAL EQUATION WITH INTEGRAL BOUNDARY CONDITION

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Abstract. The generalized quasilinearization technique for a first order nonlinear differential equation with integral boundary condition is discussed. A monotone sequence of approximate solutions converging uniformly and quadratically to a solution of the problem has been presented.

Keywords. Generalized quasilinearization, Integral boundary condition, Quadratic convergence.

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1 Introduction

The method of generalized quasilinearization introduced by Lakshmikantham [3-4] has been applied to a variety of problems [1,2,5-9]. In this paper, the method of generalized quasilinearization has been developed for a first order nonlinear differential equation with integral boundary condition. A monotone sequence of approximate solutions converging uniformly and quadratically to a solution of the problem has been obtained.

2 Preliminaries

Consider

$$\begin{aligned}x'(t) &= f(t, x), \quad t \in [0, T], \quad T > 0, \\x(0) &= \alpha x(T) + \int_0^T b(s)x(s)ds + k,\end{aligned}\tag{2.1}$$

where $f \in C([0, T] \times R, R)$, $b \in C([0, T], R_+)$, $R_+ = [0, \infty]$, $\alpha \geq 0$ and $k \in R$.

We know that the solution of the linear problem