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One Approach to Analysis of Asymptotic and Oscillation Properties of Delay and Integral PDE

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Abstract. In this paper we propose an approach to the study of oscillatory and asymptotic properties of solutions to different types of partial functional differential boundary-value problems. An analog of the classical Sturm separation theorem for PDE is presented. Zones of solutions positivity are estimated. Various assertions on the instability of PDE with memory are proved. It is demonstrated that introducing a delay in a classical heat equation, one can essentially change the oscillatory and asymptotic properties of solutions. Namely, for the respective zones, the maximum principle becomes invalid, all solutions oscillate instead of being positive, unbounded solutions appear, while all solutions of the classical Dirichlet problem tend to zero on infinity.
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1 Introduction

We consider the following equation with memory

$$Lv(\cdot,x)(t)+Tv^{''}_{xx}(\cdot,x)(t)=0, \quad x\in[0,\omega],\, t\in[0,+\infty), \tag{1.1}$$

where $L : D^n_{loc}[0,\infty)\to L_{loc}[0,\infty)$ and $T : L_{loc}[0,\infty)\to L_{loc}[0,\infty)$ are linear Volterra operators. $D^n_{loc}[0,\infty)$ is a space of k-dimensional vector functions $z : [0,\infty)\to R^k$ absolutely continuous with their derivative of $(n-1)$-th order on each finite interval, and $L_{loc}[0,\infty)$ is a space of k-dimensional locally summable vector functions $w : [0,\infty)\to R^k$. It should be noted that the operators L and T act on $v(\cdot,x)$ or $v''_{xx}(\cdot,x)$, respectively, as on functions of a variable t only for a fixed x. It is also assumed that the operators L and T cannot depend on x and do not include derivatives in x.

Definition 1.1. *A k-dimensional vector function $v : [0,\infty)\times[0,\omega]\to R^k$ is called a solution of equation (1.1) if it satisfies this equation for $x\in[0,\omega]$ and almost all $t\in[0,\infty)$, and the following conditions are met: $v(\cdot,x)\in D^n_{loc}[0,\infty)$ and $v''_{xx}(\cdot,x)\in L_{loc}[0,\infty)$ for each $x\in[0,\omega]$, $v(t,\cdot)$ and $v''_{xx}(t,\cdot)$ are continuous in x for each $t\in[0,\infty)$.*