

## Generalized Second Derivative Method and Stability Criteria for Impulsive Differential Systems <sup>1</sup>

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**Abstract.** In this paper we study stability, practical stability and eventual stability properties in terms of two measures for impulsive differential system employing Lyapunov function's generalized second derivative method.

**Keywords.** Impulse, Differential system, Lyapunov functions, Stability, Generalized second derivative method.

**AMS (MOS) subject classification:** 34A37, 93D05, 93D30.

### 1 Introduction

It is well known that the impulsive differential system has strong applied background and it has been followed with interest extensively for a long time[1]. Stability theory employing discontinuous Lyapunov function, whose derivative may be nonnegative, has recently been developed for the impulsive differential system. Nonetheless, conditions on Lyapunov function must be placed to ensure that its growth is not too rapid. Such conditions are exerted on the discrete and continuous portions of the system separately in [2]. Further the paper [3] seeks conditions that are expressed in terms of the combined discrete and continuous portions to investigate stability of the trivial solution for an impulsive differential system. But the price to pay for this type of estimate is to place a condition on the generalized second derivative of the Lyapunov function. For convenience, we call this method Lyapunov function's generalized second derivative method. Using this method, we need't consider the symbol of the first derivative of Lyapunov function. Thus when we can't determine whether the symbol of the first derivative of Lyapunov function is negative or positive, but know its generalized second derivative's symbol, this method is effective.

In this paper we consider some properties in terms of two measures for the impulsive differential systems by Lyapunov function's generalized second derivative method. It is organized as follows. Firstly we give some basic notions and concepts in section 2. Then in section 3 we establish several stability criteria. In section 4 some results on practical stability are obtained.

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