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Oscillation for Higher Order Superlinear Delay Difference Equations with Unstable Type[1]

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Abstract. This paper investigates the oscillatory behavior of the following higher order superlinear delay difference equation with unstable type

$$\triangle^m x_n = p_n \mid x_{n-k} \mid^{\alpha-1} x_{n-k}, \quad n \geq n_0, \tag{$*$}$$

where $\alpha > 1$ and m is an even integer. The existence of unbounded and nonoscillatory solution for superlinear Eq.$(*)$ is proved. Then an almost sharp criterion for bounded oscillation and nonoscillation is obtained.

Keywords. Superlinear, delay difference equation, oscillation, nonoscillation, unstable type.
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1. INTRODUCTION

Recently, there have been many investigations into the study of delay difference equations. In particular, an extensive literature now exists on the oscillation theory for delay difference equations, and various applications have been found. We refer to [1-8] and the references cited therein for more details.

Consider the delay difference equation of the form

$$\triangle^m x_n = p_n \mid x_{n-k} \mid^{\alpha-1} x_{n-k}, \quad n \geq n_0 \tag{1.1}$$

where $m \geq 2$ is an even integer, $\{p_n\}$ is a sequence of nonnegative numbers and k, n_0 are some positive integers. When $\alpha = 1$, some interesting oscillation criteria have been obtained in $[2-4]$. However, to the best of our knowledge, there is hardly any results on oscillation for Eq.(1.1) when $\alpha \neq 1$. In this paper, we will discuss the oscillation for the superlinear Eq.(1.1) when $\alpha > 1$. We first prove that Eq.(1.1) always has an unbounded positive solution.Therefore, for Eq.(1.1) we only need to find conditions for all bounded solutions to be oscillatory. For this case, we obtain an almost sharp sufficient condition which guarantees all solutions of Eq.(1.1) oscillate or Eq.(1.1) has a nonoscillatory solution.

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