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FURTHER RESULTS ON LYAPUNOV FUNCTIONS AND DOMAINS OF ATTRACTION FOR PERTURBED ASYMPTOTICALLY STABLE SYSTEMS

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Abstract. We present new theorems characterizing robust Lyapunov functions and infinite horizon value functions in optimal control as unique viscosity solutions of partial differential equations. We use these results to further extend Zubov's method for representing domains of attraction in terms of partial differential equation solutions.
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1 Introduction

The theories of Lyapunov functions and domains of attraction form the basis for much of current work in stability theory (cf. [2, 6, 8, 18]). An important result in this area is the *Zubov method* (cf. [1, 9, 11, 23, 24]), which gives conditions under which the domain of attraction of an asymptotically stable fixed point of $\dot{x} = f(x)$ is $v^{-1}([0,1))$, where v is the solution of the *Zubov equation*

$$Dv(x) \cdot f(x) = -H(x)[1 - v(x)]\sqrt{1 + ||f(x)||^2}, \quad x \in \mathbb{R}^N$$

for suitable functions H. In [5, 6, 8], Zubov's method was extended to the important case of perturbed asymptotically stable systems $\dot{x} = f(x,a)$ for which the fixed point 0 is stable under any perturbation a. These perturbations, taken to be $\mathcal{A} := \{\text{measurable functions}\, \alpha : [0,\infty) \to A\}$ for a given compact set A, are used to represent uncertainties and exogenous effects, rather than controls. The main results in [5, 6, 8] are partial differential equations (PDE) characterizations for *robust domains of attraction* and *robust Lyapunov functions* for perturbed dynamics (cf. §2 below for the relevant definitions). Under the conditions in [6], the robust domain of attraction $\mathcal{D}_o$ for the perturbed system $\dot{x} = f(x,a)$ is $v^{-1}([0,1))$, where v is the unique bounded viscosity solution of the *generalized Zubov equation*

$$\inf_{a \in A}\{-Dv(x) \cdot f(x,a) - g(x,a) + v(x)g(x,a)\} = 0, \quad x \in \mathbb{R}^N \qquad (1)$$