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ON THE NUMERICAL COMPUTATION OF THE OPTIMAL H_2-NORM AND ITS ASSOCIATED FIXED MODES IN H_2-OPTIMIZATION

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Abstract. In this paper we revisit the optimal H_2-norm and its associated fixed modes in H_2-optimization and give some new algebraic characterizations for them. Our new characterizations can be implemented directly as a numerically reliable method for computing the semi-stabilizing solution of the related linear matrix inequality, the involved disturbance decoupling problem and its associated fixed modes, and consequently computing the optimal H_2-norm and its associated fixed modes.
Keywords. H_2-optimization, H_2-norm, linear matrix inequality, numerical method, fixed mode.

1 Introduction

Unless noted, in this paper, the open left half complex plan, the imaginary axis, the open right half complex plan and the Moore-Penrose inverse of any matrix M are denoted by $\mathbf{C}^-$, $\mathbf{C}^0$, $\mathbf{C}^+$ and M^+, respectively.

H_2 optimal control problem , which contains the classical linear quadratic Gaussian (LQG) control problem as a special case, has been studied extensively in the last three decades. In particular, a complete solution to the general H_2 optimal control problem and a variety of aspects associated with it have been solved recently. Necassary and sufficient conditions, under which the optimal H_2-norm of the associated transfer function can be achieved, are given for the first time in [13]. Moreover, [14, 12] provide a thorough treatment of the H_2 optimal control problem. The works in [14, 12] include a subset of all H_2 optimal controllers, parameterizing all associated fixed modes.

Consider a linear system of the form

$$\dot{x} = Ax + Bu + Gd, \; z = Cx + Du, \tag{1}$$

where $A \in \mathbf{R}^{n\times n}, B \in \mathbf{R}^{n\times m}, G \in \mathbf{R}^{n\times q}, C \in \mathbf{R}^{p\times n}, D \in \mathbf{R}^{p\times m}$, $x \in \mathbf{R}^n$ is the state, $u \in \mathbf{R}^m$ is the input, $d \in \mathbf{R}^q$ is the disturbance, and $z \in \mathbf{R}^p$ is the output. When a feedback of the form

$$u = Fx, \tag{2}$$