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RAPID, FOURTH ORDER ACCURATE SOLUTION OF THE STEADY NAVIER STOKES EQUATIONS ON GENERAL REGIONS

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Abstract. We present a fast, new method for solving the Navier Stokes equations for an incompressible viscous fluid on an arbitrarily shaped region in two dimensions when no body forces are present. The iterative procedure we use to solve the Navier Stokes equations gives rise to an inhomogeneous biharmonic equation at each step, which we solve using an integral equation formulation. This, in turn, requires the solution of a well conditioned integral equation and the evaluation of certain surface and volume integrals. We use a Nystrom method to solve the integral equation, and a very rapid fourth order accurate finite difference method we have developed to evaluate the integrals. Computational results are provided.
Keywords. Navier Stokes Equations, integral equations, finite difference method
AMS (MOS) subject classification: 65E05, 65R20.

1 Introduction

In this paper we present a rapid new method for solving the steady Navier Stokes equations for a two dimensional viscous incompressible fluid in a general region when the Reynolds number is small and no body forces are present. Our method combines an integral formulation of the fluids problem with a fast, fourth order accurate finite difference method for evaluating the integrals that arise in the formulation.

Because the Navier Stokes equations are nonlinear they must be solved by an iterative procedure. Since we are only interested in solving problems with low Reynolds number, we use Picard iteration At each step this reduces the solution of the fluids problem to the solution of an inhomogeneous biharmonic equation

When using an integral formulation to solve an inhomogeneous problem we must first evaluate a particular solution. That is, we must first evaluate the volume integral of the product of the inhomogeneous term and a fundamental solution of the biharmonic equation. In order to evaluate such an integral we use a rapid fourth order accurate difference method we have developed. It is an extension of our method [7,13] for evaluating fundamental solutions of Poisson's equation on general regions.