

## OPTIMAL CONTROLLER FOR LINEAR SYSTEMS WITH TIME DELAYS IN INPUT AND OBSERVATIONS

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**Abstract.** This paper presents the optimal controller for unobserved linear system states with input delay, linear observations with delay confused with white Gaussian noises, and a quadratic criterion. The optimal controller equations are obtained using the separation principle. Performance of the obtained optimal controller is verified in the illustrative example against the best linear controller available for linear systems without delays. Simulation graphs and comparison tables demonstrating better performance of the obtained optimal controller are included.

**Keywords.** optimal control, filtering, linear system, time delay

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## 1 Introduction

Although the optimal controller problem for linear system states was solved in 1960s, based on the solutions to the optimal filtering [16] and regulator [21, 13] problems, the optimal controller problem for linear systems with input and observation delays has not been solved in a closed form due to the absence of closed-form solutions to the corresponding filtering and control problems for those systems. The optimal regulator problem for linear systems with state and input delays was intensively studied using various approaches (see [12, 2, 9, 4, 29, 1] and references therein); however, its solution in a closed form similar to the optimal regulator problem solution for linear systems without delays [21, 13] has not been obtained. Such complete reference books in the area as [19, 20, 23, 10, 8] note, discussing the maximum principle [18] or the dynamic programming method [26] for systems with delays, that finding a particular explicit form of the optimal control function might still remain difficult. A particular form of the criterion must be also taken into account: the studies mostly focused on the time-optimal criterion (see the paper [25] for linear systems) or the quadratic one [12, 9, 29]. The optimal filtering problem for time delay systems received not so much attention (the papers [3, 17, 15, 22] could be mentioned to make a reference), and its closed form solution similar to the