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PERIODIC SOLUTIONS OF NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS WITH INFINITE DELAY

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Abstract. In this paper Horn's fixed–point theorem is used to establish a new criterion for the existence of periodic solutions of neutral functional differential equations with infinite delay. It has been proved that if the solutions are equi–ultimate bounded, then there exists at least one periodic solution. Some known results are improved and generalized, including the famous Yoshizawa's theorem.

Keywords. Periodic solution, neutral functional differential equation, infinite delay, equi–ultimate bounded.

AMS (MOS) subject classification: 34K40, 34C25.

1 Introduction

The existence of periodic solutions of functional differential equations (hereafter, FDE), which has been studied extensively, has a theoretical as well as practical significance. A classical theorem due to Yoshizawa [13] shows that if the solutions of a retarded functional differential equation (hereafter, RFDE) with finite delay are uniformly and uniformly ultimately bounded, then it has an ω–periodic solution provided $r \leq \omega$, where r is the time delay and ω is the period of the equation. In recent years, many authors have generalized Yoshizawa's theorem. For example, the restriction $r \leq \omega$ has been removed successfully in [1,8]; it has been extended to RFDE with infinite delay by Arino, Burton and Haddack [1], and Wang and Huang [11,12]; Gao [4] has generalized to neutral functional differential equations of D–operator type (hereafter, NFDE (D, f)) with finite delay, whereas Shi [10] has extended to NFDE(D, f) with infinite delay.

A remarkable property which is due to Kato [7] is the fact that the uniformly ultimate boundedness (hereafter, UUB) does not imply the uniform boundedness (hereafter, UB). Thus, a very interesting and meaningful question can be posed as follows: is it possible that UUB without UB still guarantee the existence of periodic solutions? Burton and Zhang [2,3] gave an