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EXISTENCE AND MULTIPLICITY RESULTS OF POSITIVE SOLUTIONS FOR EMDEN-FOWLER TYPE SINGULAR BOUNDARY VALUE SYSTEMS

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Abstract. We study the existence and multiplicity of positive solutions for generalize Emden-Fowler type singular boundary value systems. We give an existence result for Dirichlet Emden-Fowler system and existence, nonexistence and multiplicity result for two point Emden-Fowler system. We see that the results may vary mainly due to boundary conditions.

Keywords. Singular boundary value system, positive solution, upper solution, lower solution, fixed point index.
AMS (MOS) subject classification: 34A37, 34B15

1 Introduction

In this paper, we consider the existence and multiplicity of positive solutions for second order systems of the form

$$(S_T) \qquad \begin{cases} u''(t) + \lambda q_1(t) f(u(t), v(t)) = 0, \\ v''(t) + \mu q_2(t) g(u(t), v(t)) = 0, \quad t \in (0,1), \\ u(0) = a, \ u(1) = b \text{ and } v(0) = c, \ v(1) = d, \end{cases}$$

where $a, b, c, d \geq 0$, λ, μ nonnegative real parameters, $f, g \in C(\mathbf{R}_+^2, \mathbf{R}_+)$ and $q_i \in C((0,1),(0,\infty))$ may be singular at $t = 0$ and/or 1. We denote $\mathbf{R}_+ = [0, \infty)$, $\mathbf{R}_+^2 = \mathbf{R}_+ \times \mathbf{R}_+$ and $\mathbf{R}_0^2 = \mathbf{R}_+^2 \setminus \{(0,0)\}$. For one dimensional scalar equations, existence and multiplicity of positive solutions for (S_T) have been studied by several authors([1] $\sim$ [5],[7],[8],[11]$\sim$ [15]). Recently, for systems, Lee[9] studied generalized Gelfand type Dirichlet boundary value systems *i.e.* $f(0,0) > 0, g(0,0) > 0$ and $a = b = c = d = 0$. Under assumptions
(H) $\int_0^1 s(1-s) q_i(s) ds < \infty$,
(H') f and g are nondecreasing on $\mathbf{R}_+^2$,
i.e. $f(u_1, v_1) \leq f(u_2, v_2)$ and $g(u_1, v_1) \leq g(u_2, v_2)$ whenever $(u_1, v_1) \leq (u_2, v_2)$, where the inequality on $\mathbf{R}_+^2$ can be understood componentwise.
(H_1) $f_\infty \triangleq \lim_{(u,v)\to\infty} \frac{f(u,v)}{u+v} = \infty, \ g_\infty \triangleq \lim_{(u,v)\to\infty} \frac{g(u,v)}{u+v} = \infty$,
he proved that there exists $(\lambda^*, \mu^*) > (0,0)$ such that problem (S_T) has