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Global Existence and Blow-up for a Periodic Integrable Shallow Water Equation with Linear and Nonlinear Dispersion

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Abstract. We establish the local well-posedness for a periodic integrable shallow water equation with linear and nonlinear dispersion, and we derive a precise blowup scenario and give an explosion criterion for strong solutions to the equation. Moreover, we prove that the equation has strong solutions which exist globally in time, provided that the initial data satisfy certain sign conditions or they are sufficiently small.
Keywords. Local well-posedness, global existence of solutions, blow-up of solutions, a periodic integrable shallow water equation, blowup scenario, an explosion criterion.
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1 Introduction

Recently, Dullin, Gottwald, and Holm [16] derived a new equation describing the unidirectional propagation of spatially periodic surface waves on a shallow layer of water

$$\begin{cases} u_t - \alpha^2 u_{txx} + c_0 u_x + 3uu_x + \gamma u_{xxx} = & \alpha^2(2u_x u_{xx} + uu_{xxx}), \\ & t > 0,\ x \in \mathbb{R}, \\ u(0,x) = u_0(x), & x \in \mathbb{R}, \\ u(t,x+1) = u(t,x), & t \geq 0, \quad x \in \mathbb{R}. \end{cases} \tag{1.1}$$

Here the constants α^2 and $\frac{\gamma}{c_0}$ are squares of length scales, and $c_0 > 0$ is the linear wave speed for undisturbed water, $u(t,x)$ stands for the fluid velocity. The equation, which is derived by the method of asymptotic analysis and a near-identity normal form transformation from water wave theory, combines the linear dispersive of the KdV equation with the nonlinear/nonlocal dispersion of the Camassa-Holm equation. It is completely integrable [16].

With $\alpha = 0$ in Eq.(1.1) we find the well-known Korteweg-de Vries equation which describes the unidirectional propagation of waves at the free surface of shallow water under the influence of gravity. $u(t,x)$ represents the wave height above a flat bottom, x is proportional to distance in the direction of propagation and t is proportional to elapsed time. The Cauchy problem of the KdV equation has been studied extensively, (see [2], [3], [20], [21], [23]) as