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Generalized Quasilinearization for Systems of Nonlinear Differential Equations with Periodic Boundary Conditions

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Abstract. In this paper, we extend the method of generalized quasilinearization to systems of nonlinear differential equations with periodic boundary conditions. More specifically, we construct, under suitable conditions, monotone sequences of lower and upper approximate solutions which converge uniformly and quadratically to the unique solution of the periodic boundary problem.

Keywords. Generalized quasilinearization, upper and lower solutions, periodic boundary value problem.

AMS (MOS) subject classification: 34A34, 34B, 37M.

1 Introduction

The original method of quasilinearization due to Bellman and Kabala [1, 2] was developed to provide a uniform approach to the study of the existence and uniqueness of the solutions of nonlinear initial-value problems (IVPs) and nonlinear boundary-value problems (BVPs), as well as to provide a uniform approach to the numerical solutions of such problems. Basically, the method consists of using linear approximation techniques together with an assumption of convexity on the function involved to obtain a sequence of lower estimates of solutions of IVPs and BVPs. The approximations are carefully constructed to be solutions of linear equations and to yield monotonicity and rapid convergence. These properties which are most important in computational work make the method of quasilinearization highly attractive not only in theoretical studies but in applications as well [2].

However, the convexity on the function involved turns out to be a major drawback as it limits the applicability of the method. This was recognized by Lakshmikantham who first generalized the method by lifting the convexity requirement [3, 7] and then extended it using the method of lower and upper solutions together with the monotone iterative technique to simultaneously construct monotone sequences of both lower and upper bounds of the solutions. The bounds themselves are solutions of linear equations. All this is