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THEOREMS OF PERRON TYPE FOR UNIFORM EXPONENTIAL STABILITY OF LINEAR SKEW-PRODUCT SEMIFLOWS

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Abstract. The aim of this paper is to give a unified treatment for the theorems of Perron type for uniform exponential stability of linear skew-product semiflows. First, we present necessary and sufficient conditions for exponential stability of discrete linear skew-product semiflows. As a consequence we give an estimation for the stability radius of a discrete linear skew-product semiflow. After that, the results are extended for the general case of linear skew-product semiflows. Then, we obtain characterizations for uniform exponential stability of strongly continuous linear skew-product semiflows in terms of Banach function spaces, generalizing some stability theorems for evolution operators due to Datko, Neerven, Minh, Räbiger, Schnaubelt, Clark, Latushkin, Montgomery-Smith and Randolph.
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1 Introduction

The input-output criterions in the study of the asymptotic behaviour of evolution equations, have an impressive history that started with the work of Perron ([29]). Important extensions of Perron's ideas to the infinite dimensional case have been presented by Massera and Schäffer ([17]) and by Daleckii and Krein ([10]). Recently, remarkable results of Perron type, for stability, expansiveness and dichotomy, respectively, have been obtained in [2], [4], [8], [16], [18], [21]-[28], [30]. The case of uniform exponential stability of evolution operators has been treated in [8], [11], [25], [26]. For an evolution operator $\mathcal{U} = \{U(t,s)\}_{t\geq s\geq 0}$ on a Banach space X one defined the operator $\mathcal{P}f := P_f$, where

$$P_f(t) = \int_0^t U(t,s)f(s)\,ds$$

for all $f \in L^1_{\rm loc}(\mathbf{R}_+, X)$ and all $t \geq 0$. Roughly speaking, the uniform exponential stability of an evolution operator $\mathcal{U} = \{U(t,s)\}_{t\geq s\geq 0}$ has been expressed in terms of boundedness of the operator $\mathcal{P}$ on $C_0(\mathbf{R}_+, X)$ and $L^p(\mathbf{R}_+, X)$, respectively. The particular case of C_0 - semigroups has been