Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **12 (2005)** 139-150

STABILITY INVARIANCE OF A PERIODIC LINEAR SWITCHED SYSTEM

J. M. Almira[1], N. Del-Toro[1], and P. J. Torres[2]

[1]Departamento de Matemáticas
Universidad de Jaén. E.U.P. Linares, Linares (Jaén), Spain 23700
[2]Departamento de Matemática Aplicada
Universidad de Granada, Granada, Spain 18071

Abstract. In this paper we study the stability character of the linear differential equation $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where $\mathbf{A} = \mathbf{A}(t)$ is a piecewise constant matrix function of period $T > 0$, with $\mathbf{A}(t) \in \mathcal{C}_n$ for all t and a fixed class $\mathcal{C}_n$ of matrices of order n. Concretely, we are interested in the characterization of the permutations of the pieces of $\mathbf{A}$ which do not change the stability character of the equation. We completely solve the problem for a generalized version of Meissner equation which is also of interest from the physical point of view.
Keywords. Stability, Meissner's equation, Periodic Linear Switched system, Dahlquist condition, Spectrum, Groups of Permutations.
AMS (MOS) subject classification: 34K20, 34K45, 35B35

1 Introduction

Let $\mathcal{C}_n$ be a certain fixed class of matrices of order $n \in \mathbb{N}$, and let $\mathbf{A}(t)$ be a piecewise-constant periodic matrix function of period $T > 0$,

$$\mathbf{A}(t) = \mathbf{A}_i,\ t \in \mathbf{I}_i = (h(i-1), hi];\ \ \mathbf{A}_i \in \mathcal{C}_n \text{ for all } i \le N; \tag{1}$$

where $Nh = T$. Let us consider the linear system of differential equations

$$\mathbf{x}'(t) = \mathbf{A}(t)\mathbf{x}(t). \tag{2}$$

Such a system is a special case of the so-called *switched systems*. A switched system is typically defined by a family of continuous systems and a *switching signal* describing the jumps between them (in our case, the switching signal is periodic). This topic has become very popular, specially in the frame of control theory, and a considerable amount of references are avaiable (see for instance [1,3,9,10,14,15] only to mention some of them). In applications, switched systems arise in a natural way from processes which present abrupt changes of the conditions. In this sense, the survey [10] presents an extensive bibliogaphy with big number of applications.

We say that (2) is stable if all its solutions are bounded and we say that it is asymptotically stable if

$$\lim_{t\to\infty} ||x(t)|| = 0$$