

## Global optimization for a special class of discrete-valued optimal control problems

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**Abstract.** A special class of optimal discrete-valued control problems is solved by two different global optimization techniques, namely, the Simulated Annealing algorithm (SA) and the Genetic algorithm (GA). Two numerical examples have been solved to examine the efficiency of each of these two methods.

**Keywords.** Discrete valued, optimal control, global optimization, simulated annealing, genetic algorithm.

## 1 Introduction

In many practical engineering applications, the control action is restricted to a set of discrete values. A simple example might involve an on-off switch. Other examples include switched amplifier designs [13] and optimal driving strategies for trains [6].

The optimal control problem considered in this paper is almost the same as that considered in [9]. It involves a cost functional which is to be minimized over a class of discrete-valued controls subject to a dynamical system together with some terminal state constraints. The locations of the discontinuities of the piecewise constant control functions are referred to as switching points. For most practical problems, the optimal solution involves only a finite number of switching points. As mentioned in [7], the difficulty with these problems is that the range set of the control is discrete and hence not convex. Furthermore, choosing the appropriate elements from the control set in an appropriate order is, in fact, a non-linear combinatorial optimization problem.

In [9], the control parameterization enhancing technique (CPET) is used to transform the problem into an equivalent optimal parameter selection problem, which can be solved efficiently by the software MISER3 reported in