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Solutions of a Pair of Conservation Laws with Nonconvex Flux Function

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Abstract. We consider a pair of hyperbolic conservation laws which is a model for the motion in one-dimensional nonlinear elestic body. Our model is similar to the isentropic Euler system but allows dependent variables to vary from negative to positive values. So the flux function is nonconvex. The Riemann problem for it is solved by the classical solutions, *i.e.* the combinations of a shock, a rarefaction wave and a rarefaction-shock. The numerical study on an initial wave with positive and negative values showed that the positive part moves to the right and the negative part moves to the left. They collide and cancel each other, but recover after a time. The recovered parts form discontinuous shocks and continue to go in their respective directions. The asymptotic decay rate of mechanical energy is evaluated by applying the Temple's formula for the decay of solutions.
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1 Introduction

We consider the motion in one-dimensional elastic body where adjacent material particles interact with a potential V. The potential is assumed to be a smooth function of the deformation gradient $U_x(x,t)$ at the point x and time t, $V(U_x(x,t))$. Then the equation of motion is,

$$U_{tt} = (V'(U_x))_x. \tag{1}$$

From the symmetry of motion, $V''(U_x) > 0$. If we denote the location of the material particle initially at x by $y(x,t)$, the deformation is given by $U(x,t) = y(x,t) - x$. From the impenetrability of matter [1], $y(x,t)$ should be a strictly increasing function of x, and so the range of deformation gradient is restricted to $U_x(x,t) > -1$.

The smooth solutions of (1) conserve the total mechanical energy $E(t)$,

$$E(t) = \int_{-L}^{L} \left[\frac{1}{2}U_t^2 + V(U_x)\right] dx, \tag{2}$$