*Dynamics of Continuous, Discrete and Impulsive Systems*
*Series* **B:** *Algorithms and Applications* **11 (2004)** 673-700


# RECURSIVE COMPUTATION OF NASH STRATEGY FOR MULTIPARAMETER SINGULARLY PERTURBED SYSTEMS

Hiroaki Mukaidani[1] and Hua Xu[2]

[1]Graduate School of Education,
Hiroshima University, 1–1–1, Kagamiyama,
Higashi–Hiroshima, 739–8524 Japan
[2]Graduate School of Business Sciences,
The University of Tsukuba, 3–29–1, Otsuka,
Bunkyou-ku, Tokyo, 112–0012 Japan

**Abstract.** In this paper, the linear quadratic Nash games for infinite horizon multiparameter singularly perturbed systems (MSPS) are discussed. Compared with the existing results, the nonsingularity assumption for the state matrices of the fast subsystems is not needed. The uniqueness and the asymptotic structure of the solution to the generalized cross–coupled multiparameter algebraic Riccati equations (GCMARE) are newly established without the nonsingularity assumptions. The main contribution of this paper is that a new algorithm which is based on the recursive algorithm for solving the GCMARE is proposed. It should be noted that the new equation to get the initial guess has simple form. As another important feature, the control strategies can be constructed in the same dimension of the slow and fast subsystems.
**AMS (MOS) subject classification:** 34K26, 34K28, 49N70

## 1 Introduction

The control problems for the multiparameter singularly perturbed systems (MSPS) have been investigated extensively (see e.g., [1] and reference therein). In these various studies of the MSPS, the linear quadratic Nash games for the MSPS have been studied [2,3,11]. It is well–known that the Nash equilibrium strategies are based on the solution of the cross–coupled multiparameter algebraic Riccati equations (CMARE). There exist two main approaches for constructing the Nash equilibrium strategies of the MSPS. One is the composite design method [2,3], and the other one is the numerical technique [11]. When the parameters represent the small unknown perturbations whose values are not known exactly, the composite strategies are very useful. However, the composite Nash equilibrium solution achieves only a performance which is $O(\mu)$ (where $\mu := \sqrt{\varepsilon_1 \varepsilon_2}$) close to the full–order performance. As another important drawback, since the closed–loop solution of the reduced