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RICHARDSON EXTRAPOLATION OF GALERKIN FINITE ELEMENT METHODS FOR PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. The object of this paper is to investigate Richardson extrapolation of two different schemes for finite element approximations of a linear parabolic partial differential equation with homogeneous Dirichlet boundary conditions, which can lead significantly to the improvement in the accuracy of approximations with the help of an interpolation postprocessing technique. As a by-product, we illustrate that all the approximations of higher accuracy can be used to generate efficient a posteriori error estimators.
Keywords. Parabolic partial differential equations, Galerkin finite element methods, Richardson extrapolation, interpolation postprocessing, a posteriori error estimators.
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1 Introduction

Our purpose in this paper is to study the Galerkin finite element method for a linear parabolic partial differential equation whose classical formulation is: Find $u = u(x, y, t)$ such that

$$\begin{array}{llll} u_t & = & \nabla \cdot (a(x,y,t)\nabla u) + f & \text{in } \Omega \times J, \\ u & = & 0 & \text{on } \partial\Omega \times J, \\ u(x,y,0) & = & u_0(x,y) & \text{in } \Omega, \end{array} \tag{1.1}$$

where $\Omega \subset R^2$ is an open bounded domain with a Lipschitz boundary $\partial\Omega$, $J = (0, T]$ with $T > 0$, $f \in L^2(\Omega)$ and $a(x, y, t) \geq a_0$ are known functions, and a_0 is a known positive constant.

The problem (1.1) can arise from many physical processes and the study of its numerical methods has received considerable attention in the past. For example, Thomée [19] has formulated finite element methods for the problem