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A NEW MODEL FOR A DISCHARGING, COUPLED TANK AND CAPILLARY TUBE SYSTEM

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Abstract. We introduce a novel model for the incompressible viscous fluid draining from a coupled tank and capillary tube system. By physics laws, we set up a nonlinear differential equation, which governs this model, and derive its parametric solution. Performing asymptotic analysis, we show that existing well-known models can be redeveloped from our new model. In addition, we analyze the maximum deviation between the new model and the exponential decay model. Experimental results back up our analytic analysis.
Keywords. The coupled tank and capillary tube system, parametric model, exponential decay model, deviation analysis, asymptotic approximation.
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1 Introduction

Modeling incompressible viscous fluid draining from a tank is a common, introductory exercise in fluid mechanics. There are two well-known models. The first type of draining tank presented in most introductory physics texts (for example, see Reese [4]) is a simple cylinder with a hole punched in the bottom. This model was derived by E. Torricelli in 1640. The second type of a draining tank, one in which a long horizontal capillary tube is attached to the drain, now includes the effect of viscosity. Assuming that the radius of the capillary tube is much smaller than the length of the capillary tube, Smithson and Pinkston [2] have shown that the height of fluid level in the tank decays exponentially. This model is widely accepted due to its simplicity and its similarity with other real-life phenomena such as heat transfer, radioactive decay, and pumping down a vacuum system as illustrated by Ruby [1]. Represented purely as exponential decay, such mathematical models often neglect the kinetic energy of the draining fluid for the sake of convenience. However, by retaining the kinetic energy, we can develop a better model of the fluid height in the tank as a function of time. Our analysis introduces a nonlinear differential equation that can be used to model multiple types of draining systems with and without viscosity. Further, we derive the parametric solution for the nonlinear differential equation. By doing the asymptotic