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DERIVATIVE SUPERCONVERGENCE OF LINEAR FINITE ELEMENTS BY RECOVERY TECHNIQUES

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Abstract. The aim of this article is to investigate the superconvergence in derivative approximations of finite element solutions. We construct three kinds of derivative recovery formulas at the mesh points for linear, bilinear and quadrilateral finite elements, respectively, in the approximations of second order elliptic boundary value problems. These recovery formulas are simpler and more available comparing to the existing formulas. We also show the superconvergence for each derivative recovery formulas.
Keywords. boundary problems, finite element, derivative approximations, recovery techniques, superconvergence.
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1 Introduction

Linear finite elements are one kind of the most common finite elements used in the science and engineering calculations. But the derivatives of linear finite element solutions are discontinuous at the mesh nodal points, we can not directly calculate the nodal point values of derivative. In order to overcome this difficulty and obtain high accuracy derivative approximation, in recent years, some derivative recovery techniques are proposed, for example, the patch recovery technique by Zienkiewicz and Zhu [1, 2], the interpolating post processing technique by Lin and Zhu [3], and the derivative patch interpolation recovery technique by Zhang [4], etc.

In this paper, we will establish three kinds of derivative nodal point recovery formulas for the linear triangular element, bilinear rectangular element and quadrilateral element, respectively, in the finite element approximations for second order elliptic boundary value problems. These recovery formulas can be used to calculate the derivative values of finite element solutions at mesh nodal points, and possess the superconvergence properties. Comparing to the known derivative recovery techniques, our methods are simpler and more available, with higher accuracy, and the computational cost of our methods is almost free.

Let $\Omega \subset R^2$ be a convex polygonal domain, $J_h = \{e\}$ be the finite element partition of domain Ω, e the partition element, h the partition diameter, and