Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **11 (2004)** 615-630

FULLY DISCRETE POSTPROCESSING GALERKIN METHOD FOR THE NAVIER-STOKES EQUATIONS

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Abstract. In this paper we consider a fully discrete postprocessing Galerkin method for the nonstationary Navier-Stokes equations with non-smooth initial data and time-dependent body force. Here the postprocessing technique uses approximate inertial manifolds under time discretization to approximate the high modes (the small scale components) in the exact solution under time discretization in terms of the Galerkin approximations, which in this case play the role of the lower modes (large scale components). Moreover, we analyze the approximate accuracy of the numerical solutions. The theory analysis shows that the new method is a very efficient algorithm for improving the accuracy of the standard Galerkin method with very little extra computational cost.
AMS (MOS) subject classification:35Q10, 65N30, 65P25, 76D05.

1 Introduction

The purpose of this paper is to present a fully discrete postprocessing Galerkin method which is an extension and improvement of the postprocessing Galerkin method on the spatial discretization (see [4,5]). We consider the two dimensional nonstationary Navier-Stokes equations in an appropriate Hilbert space H (see [1, 3, 5, 12]):

$$\frac{du}{dt} + \nu Au + B(u,u) = f\,, \tag{1.1}$$

with the initial condition

$$u(x,0) = u_0(x)\,, \tag{1.2}$$

where A is the Stokes operator and B is the projection of the nonlinearity on the space of divergence-free functions. We refer to section 2 for the functional setting, as well as for some properties of the solution .

Since the inverse A^{-1} of the Stokes operator A is compact self-adjoint (see [3, 4-5, 12]), A possesses a complete family of eigenvectors w_j which is orthonormal in H

$$Aw_j = \lambda_j w_j\,, 0 < \lambda_1 \le \lambda_2 \le \cdots\,, \lambda_j \to \infty \text{ as } j \to \infty\,. \tag{1.3}$$