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GENERALIZED INVERSE EIGENVALUE PROBLEM AND SPECTRAL FUNCTION

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Abstract We consider an inverse generalized eigenvalue problem (IGEP) $Ax = \lambda Bx$. We determine a unique Jacobi matrix A via a given spectral function $\tau(\lambda)$ and a nonsingular diagonal matrix B. This is corresponding to the continuous case of the inverse problem.

1. Introduction

Let $\sigma(A)$ and $\sigma_1(A)$ denote the eigenvalues of A and A_1 where A_1 is obtained from A by deleting the first row and first column of A. It is well-known that a Jacobi matrix A, a tridiagonal matrix with positive off-diagonals, can be uniquely determined by two interlacing sequences $\{\lambda_i\}_1^n$ and $\{\mu_j\}_1^{n-1}$, where

$$\lambda_1 < \mu_1 < \lambda_2 < \mu_2 < \ldots < \mu_{n-1} < \lambda_n$$

such that $\sigma(A) = \{\lambda_i\}_1^n$ and $\sigma_1(A) = \{\mu_j\}_1^{n-1}$. In this paper we use the concept of *m-functions* to determine a unique Jacobi matrix A by defining an appropriate spectral function $\tau(\lambda)$. In the literature the usual method to determine the matrix A via given spectral data is related to orthogonal polynomials. Alaca [1] considered the IGEP whenever B is in an *indefinite* diagonal matrix, Hald [6] investigated classical inverse eigenvalue problem $Ax = \lambda x$, and Ghanbari and Mingarelli [4] discussed the IGEP $Ax = \lambda Bx$ where A and B are tridigonal matrices with B positive definite. Most of the authors (e.g. [1,2,6]) applied the theory of orthogonal polynomials (e.g. [3]) to treat the problem. A comprehensive work on spectral analysis of infinite Jacobi matrices was given by Teschl [8]. We also studied infinite dimensional case of IGEP in [5]. Using the concept of m-functions in this paper, we construct a unique Jacobi matrix A by finding a recursive relation among the entries of A.