

THE MAXIMUM RANK OF THE MATRIX PENCIL $E + \sum B_i K_i C_i$ AND ITS APPLICATIONS

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Abstract. A new equality about the maximum rank of the matrix pencil $E + \sum B_i K_i C_i$ is obtained when K_i varies in $R^{m_i \times l_i}$ ($i = 1, 2, \dots, N$). The equality may be calculated expediently. Some applications are discussed in the impulse decentralized fixed modes, the infinite decentralized fixed modes and the normalization of singular decentralized control systems for the singular decentralized control systems.

Keywords. Matrix pencil, maximum rank, singular decentralized control systems, impulse decentralized fixed modes, infinite decentralized fixed modes

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1 Introduction

At the present time, many equalities are applied widely in mathematics and control theory as well as other subjects. The equality about the maximum rank of the matrix pencil $E + BKC$ (i.e., $N = 1$), corresponding to singular centralized control systems, has been obtained when K varies in $R^{m \times l}$ in the literature [1] and applied triumphantly to the fixed modes and decentralized control for singular decentralized control systems [2, 3].

In this paper, a new equality about the maximum rank of the matrix pencil $E + \sum B_i K_i C_i$ is obtained when K_i varies in $R^{m_i \times l_i}$ ($i = 1, 2, \dots, N$). The equality may be calculated expediently. Some applications are discussed in the impulse decentralized fixed modes, the infinite decentralized fixed modes and the normalization of singular decentralized control system for the singular decentralized control systems.

2 Preliminary knowledge

Lemma 2.1 Let $P_i \in R^{m \times m_i}$, $Q_i \in R^{l_i \times n}$, $i = 1, 2, \dots, N$. If $(P_1 \ P_2 \ \dots \ P_N)$ is full rank for columns, and $(Q_1^T \ Q_2^T \ \dots \ Q_N^T)^T$ is full rank for rows, then

$$\max_{F_i} \text{rank} \left\{ \sum_{i=1}^N P_i F_i Q_i \right\} = \sum_{i=1}^n \min \{ \text{rank } P_i, \text{rank } Q_i \}. \quad (2.1)$$